SCHOOL

Trial WACE Examination, 2012

Question/Answer Booklet

MATHEMATICS
SPECIALIST 3C/3D
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words ______________________

Your name ______________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free

(50 Marks)

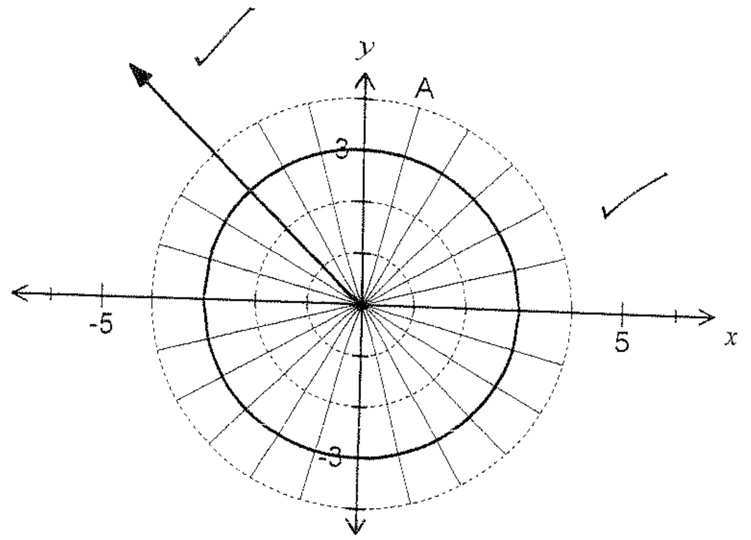
This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1

(5 marks)

(a) Sketch the polar graphs of $r=3$ and $\theta=\frac{3\pi}{4}$ on the axes below. (2 marks)



(b) A is the point with polar coordinates $\left(4,\ \frac{5\pi}{12}\right)$. Find the exact distance between A and the point of intersection of the graphs of $r=3$ and $\theta=\frac{3\pi}{4}$. (3 marks)

$$d^2 = 3^2 + 4^2 - 2\times 3\times 4\times\cos\left(\frac{3\pi}{4}-\frac{5\pi}{12}\right)$$

$$d^2 = 9+16-24\cos\frac{\pi}{3}$$

$$d^2 = 13$$

$$d=\sqrt{13}$$

Question 2 **(6 marks)**

(a) Determine $\int 12xe^{x^2}(e^{x^2}+1)^2\,dx$. (2 marks)

$u = e^{x^2} + 1$
$du = 2xe^{x^2}\,dx$ ✓

$\boxed{2(e^{x^2}+1)^3 + c}$ ✓

(b) Evaluate $\int_{0.2}^{1} \frac{9}{x^2}\sqrt{1+\frac{3}{x}}\,dx$, using the substitution $u = 1+\frac{3}{x}$ or otherwise. (4 marks)

$du = -\frac{3}{x^2}\,dx$ ✓

When $x=1$, $u=4$ and when $x=0.2$, $u=16$. ✓

$\int_{16}^{4} -3u^{\frac{1}{2}}\,du$

$= \left[-2u^{\frac{3}{2}}\right]_{16}^{4}$ ✓

$= [-2(8)] - [-2(64)]$

$= -16 + 128$

$= 112$ ✓

Question 3

(5 marks)

Determine a unit vector perpendicular to the plane of $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

Let a perpendicular vector to plane be $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + \mathbf{k}$ (assuming $\mathbf{k}$ coefficient is not 0). Then: or some other start ✓

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0 \Rightarrow x + 2y = 3$$

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0 \Rightarrow x + y = 1$$

✓✓ recognises zero dot product

$x + 2y - (x + y) = 3 - 1 \Rightarrow y = 2$ solution of equations ✓

$x = 1 - 2 = -1$

Hence $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and so required unit vector is

$$\pm\frac{1}{\sqrt{6}}(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

unit vector ✓

~~both directions~~

Question 4 **(11 marks)**

(a) Find $\frac{dy}{dx}$ for the following:

(i) $y = \log_{16}\left(x^2\right)$. (3 marks)

$$y = \frac{\ln x^2}{\ln 16} \quad \checkmark$$
$$y = \frac{2\ln x}{4\ln 2} \quad \checkmark$$
$$\frac{dy}{dx} = \frac{1}{2x\ln 2} \quad \checkmark = \frac{2}{x\ln 16}$$

(ii) $y = \cos^3(e^{(1-x^2)})$. (3 marks)

$$\frac{dy}{dx} = 3(-2xe^{(1-x^2)})(-\sin(e^{(1-x^2)})\cos^2(e^{(1-x^2)}) \quad \checkmark\checkmark$$
$$= 6xe^{(1-x^2)}\sin\left(e^{(1-x^2)}\right)\left(\cos^2 e^{(1-x^2)}\right) \quad \checkmark$$

(b) Find the equation of the tangent to the curve $y = \sqrt{x+y}$ when $y = 6$. (5 marks)

$$\frac{dy}{dx} = \frac{1}{2}\left(1 + \frac{dy}{dx}\right)\frac{1}{\sqrt{x+y}} \quad \checkmark\checkmark$$ correct differentiation

When $y = 6$, $6 = \sqrt{x+6} \Rightarrow x = \underline{\underline{30}}$ ✓ location of abscissa

$$\frac{dy}{dx} = \frac{1}{2}\left(1 + \frac{dy}{dx}\right)\frac{1}{6}$$
$$12\frac{dy}{dx} = 1 + \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{1}{11} \quad \checkmark$$ gradient

Tangent is $y - 6 = \frac{1}{11}(x - 30) \Rightarrow 11y - x = 36$ ✓ line

$$y = \frac{x}{11} + \frac{36}{11}$$

Question 5

(7 marks)

(a) Find all complex solutions of the equation $z^4 = 8 + 8\sqrt{3}i$. (5 marks)

$$z^4 = 16cis\left(\frac{\pi}{3}\right)$$
$$z_0 = \sqrt[4]{16}cis\left(\frac{\pi}{3} \times \frac{1}{4}\right)$$
$$= 2cis\left(\frac{\pi}{12}\right)$$
$$z_1 = 2cis\left(\frac{7\pi}{12}\right)$$
$$z_2 = 2cis\left(-\frac{5\pi}{12}\right)$$
$$z_3 = 2cis\left(-\frac{11\pi}{12}\right)$$

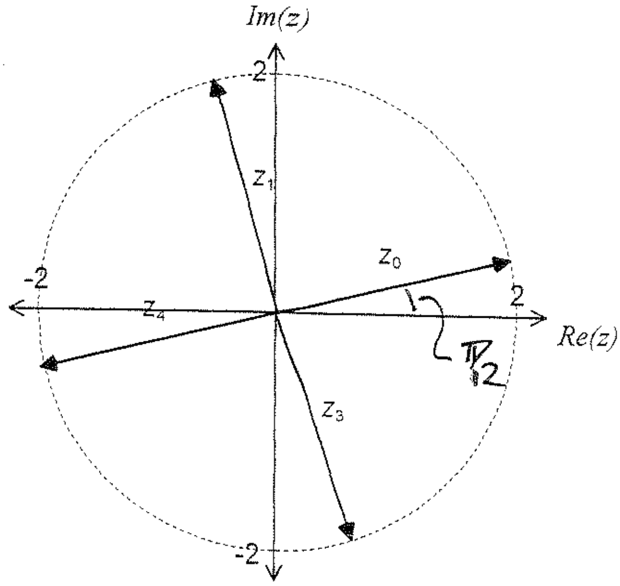
$$z^4 = 16\,cis\left(\frac{\pi}{3} + 2k\pi\right)$$
$$z = 2\,cis\left[\frac{(6k+1)\pi}{12}\right]$$

convert to cisθ form ✓

recognise periodic nature of z^4 ✓

take roots ✓✓✓

(b) Sketch the solutions from (a) on the complex plane. (2 marks)



must include ~~initial~~ reference angle

Question 6

(9 marks)

(a) Find the value(s) of a for which the matrix $\begin{bmatrix} a & 3 \\ 1 & a+2 \end{bmatrix}$ is singular. (2 marks)

Singular if

$a(a+2) - 1\times 3 = 0$

$a^2 + 2a - 3 = 0$ ✓

$(a+3)(a-1) = 0$

$a = -3,\ a = 1$ ✓✓

(b) A system of linear equations is given by

$$a + 4b + 2c = 9$$
$$2a + 3b = 13 + c$$
$$3c + 3a + 9 = 2b$$

The system can be solved using the matrix equation $Q = P^{-1}R$. Write down suitable matrices for P, Q and R, but do not solve the system. (3 marks)

$P = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & -1 \\ 3 & -2 & 3 \end{bmatrix}$ $Q = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $R = \begin{bmatrix} 9 \\ 13 \\ -9 \end{bmatrix}$

✓ ✓ ✓

(c) Find matrix A if $AB + 2B = 4I$, where I is the identity matrix and $B = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$. (4 marks)

$(A + 2I)B = 4I$

$A = 4B^{-1} - 2I$

$A = 4 \times \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

or $A = (4I - 2B)B^{-1}$ ✓✓

✓

✓

Question 7 **(7 marks)**

(a) Show that $\frac{1+e^{a}}{1+e^{-a}}=e^{a}$, where a is a constant. (2 marks)

$$\frac{1+e^{a}}{1+e^{-a}}=\frac{e^{a}}{e^{a}}\times\frac{1+e^{a}}{1+e^{-a}}$$
$$=\frac{e^{a}(1+e^{a})}{e^{a}+1}$$
$$=e^{a}$$

✓ ✓

(b) Show that $\int_{-3}^{3}\frac{e^{kx}}{1+e^{kx}}dx=3$, where k is a constant. (5 marks)

$$\frac{1}{k}\int_{-3}^{3}\frac{ke^{kx}}{1+e^{kx}}dx=\frac{1}{k}\left[\ln\left|1+e^{kx}\right|\right]_{-3}^{3}$$
$$=\frac{1}{k}\left(\ln\left(1+e^{3k}\right)-\ln\left(1+e^{-3k}\right)\right)$$
$$=\frac{1}{k}\ln\left(\frac{1+e^{3k}}{1+e^{-3k}}\right)$$
$$=\frac{1}{k}\ln\left(e^{3k}\right)$$
$$=\frac{1}{k}\times 3k$$
$$=3$$

✓ ✓ ✓ ✓ ✓

or $u = 1+e^{kx} \Rightarrow du = ke^{kx}\,dx$

$$\Rightarrow \int \frac{e^{kx}}{1+e^{kx}}\,dx = \int \frac{1}{k}\frac{du}{u}$$
$$= \frac{1}{k}\ln u + c$$
$$= \frac{1}{k}\ln\left(1+e^{kx}\right)\Big|_{-3}^{3}$$
$$= \frac{1}{k}\left[\ln\frac{1+e^{3k}}{1+e^{-3k}}\right]$$
$$= \frac{1}{k}\ln e^{3k}$$
$$= 3$$

End of questions